Illustrating Deduction. A Didactic Sequence for Secondary School

Francisco Saurí

Universitat de València. Dpt. de Lògica i Filosofia de la Ciència
Cuerpo de Profesores de Secundaria. IES Vilamarxant (España)

Francisco.Sauri@uv.es

Abstract. This paper describes author's experience in quickly (in 3-4 hours) introducing secondary students to the concept of logical deduction in a philosophy course. The method involves beginning with an argument concerning a topic of the syllabus. Focusing on the notions of premise and conclusion, without introducing the language of propositional logic, the paper emphasizes logical arguments used to obtain information that cannot be observed or communicated by an informant. Only after presenting several additional arguments of the same logical form as the first argument, and re-emphasizing the relationship between the notions of premise and conclusion, does the author informally introduce the notions of propositional variables, connectives, and logical deduction. The idea for teaching deduction in this way is not original; the purpose is illustrating deduction quickly. [1]

Keywords: deduction, argument, secondary school, K12, formal logic, didactic sequence, logical validity.

[2] The concept of deduction is very important in philosophical discussions. In the empirical knowledge, deduction is distinguished from the induction but they work linked. And deduction is central in mathematics.

However, in secondary school it can only be spent a little time on this matter. But suppose that in the course, we must explain that Plato's ideal mathematics works deductively. When I was a secondary school student, Philosophy was explained after making deductions in mathematics in the previous year. But now that does not happen. You can not say: a deduction is such a thing that you do sometimes in math class.

Now we can provide various examples of deductions and we can compare with examples of inductive reasoning. But why deduction and induction are different? We can give a definition of deduction: if the premises are true, necessarily the conclusion is true. This definition make sense for the specialist but, normally, for the students, it is empty, without reference, simple verbalism. My purpose is provide a quick solution.

In a good objection, an anonymous referee has pointed out that nothing is said here about

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[1]The author is grateful to an anonymous referee for the idea of this abstract.
[2]The author is grateful to two anonymous referees for their thoughtful comments and suggestions

assessment. But let’s consider this. All teachers have had to explain issues that, while they are not central, they are supposed partially known but some students ask about it. After the explanation, the teacher asks: do you understand me? Obviously the teacher doesn’t hope that the student has learned the concept completely but he hopes only that the student gets an idea. You can provide examples of deductions as I said but what is the way to prove its deductive character? That is my purpose in this didactic sequence.

Therefore we should select: 1) what contents are fundamental for a quick introduction, and 2) what kind of explanation we give on every content. My proposal for the former is obvious: reasoning, premises, conclusion, logical form, semantic definition of validity and truth table. More difficult is the question of the kind of explanation, and how deeply to explain such concepts. Although the first three concepts offer no difficulty, a sufficiently wide range of examples will suffice, How deep must we arrive at the explanation of the rest of concepts? This article answers this question by a didactic sequence based on my experience with students of first course of Bachillerato. Its length is 3 or 4 hours. What is the process?

In my opinion is advisable to teach this issue as a part of a philosophical problem. In recent years, my choice has been to introduce the concept of deduction in treating teleology and Aquinas' argument that God exists. As an anonymous referee has pointed out, we can doubt this chosen example would necessarily be the most suitable in very secular societies. But to begin with we ask the students to attack or defend this argument and they love controversy. The result will be that if we want to put the defender in trouble, we can only attack the truth of the premises. The argument is well made. In terms of logic, is formally valid. But we must prove to be so.

To do this in a second stage, we will offer various arguments that deviate from the topic of the first argument, but they have the same force and structure, the same logical form. It is advisable to introduce the arguments in dialogue form because a text in dialog form limits clearly the relevant information. Some students could state that has not been taken into account all information, all the premises. But the situation puts the limits.

The third stage serves to underwrite the conditions under which the arguments are valid. We must drive the attention of students to the intuitive fact that the examples discussed lead to indisputable conclusions once accepted the premises (the accepted information). This is not to advance the semantic definition of validity and apply it. We must show an intuition and only afterwards we can do the definition.

From here, we will show that this intuition has a rigorous demonstration (fourth stage). We need some student knowledge about grammar, about compound propositions, and ask them about the meaning of certain propositions connected by words like “and”, “or” and “no”. We drive their explanations to the truth role of these words: the truth table of them is easy by that means.

In the last fifth stage, we will use that knowledge and it will be applied to the semantic definition of validity: when all the premises, at once, are true then the truth of the conclusion is guarantee.

We prove this by means of a truth table.

First stage. The context.

How to introduce and illustrate the concept of deduction in the subject of Philosophy? My choice is to do it within a matter of course that raises the validity of an argument. In particular I have chosen the matter of teleology and the teleological proof of God. Normally, I begin my class with this sentence: I can prove God exists.

I use the version of Thomas Aquinas in his *Summa Theologica.* It reads:

[1] "The fifth way is taken from the governance of the world. We see that things which lack intelligence, such as natural bodies, act for an end, and this is evident from their acting always, or nearly always, in the same way, so as to obtain the best result. Hence it is plain that not fortuitously, but designedly, do they achieve their end. Now whatever lacks intelligence cannot move towards an end, unless it be directed by some being endowed with knowledge and intelligence; as the arrow is shot to its mark by the archer. Therefore some intelligent being exists by whom all natural things are directed to their end; and this being we call God."
Thomas Aquinas, *SummaTheologica,* Part 1, Question 2, Article 3.

The argument needs a lot of explanation. This is used to introduce the concept of teleology. But that is not my concern now. Then we will jump to the result of the textual analysis. After analysis, the argument would look like this:

[1']
[premise 1] The natural bodies act intentionally (q). [premise 2] The natural bodies lack of intelligence (not r). [premise 3] If natural bodies lack intelligence (not r) then they did not act intentionally (not q), if they are not directed by someone intelligent (not d). [conclusion] Then there is an intelligent being who directs all natural things to an end (d).

And that being can be identified with God as suggested by Aquinas himself.

At this point is worth remembering that reasoning is presented in various ways, and that there is an important difference between premises and conclusion. This can not be neglected, otherwise, students may not know what we mean. (Previous **exercises** are needed here.)

Besides, my recommendation is not to introduce the propositional variables brought here to facilitate the work of teachers. They will be used later.

To understand the design argument and introduce the similarities between this argument and other arguments we can introduce an argument like [2].

[2]
Dick: "The failure of all systems of the spacecraft could only have been intentional."(q)

[premise1]
Tom & Dick: "There is a saboteur on board!"

That is, [conclusion] someone has tampered with the spacecraft systems (d) and he is called saboteur.

Both speakers assume the following premises:
[premise 2]: "the spacecraft systems can not reach an agreement to fail at once", which is the equivalent of "natural things lack intelligence" (not r)

[premise 3] "If the systems of the spacecraft can not reach an agreement to fail at once (not r) then the failure of all systems can not have been intentional (not q) if they are not directed by someone (not d). "

The aim of this argument [2] is to point out how you can run a similar argument in a context that is also seeking intentions. To signal their similarities, let us see [2] in a paragraph (another **exercise** for students):

[2 ']
[premise 1] Spacecraft systems have performed intentionally (q). [premise 2] The spacecraft systems lack intelligence (not r). [premise 3] If systems lack intelligence (not r), then they do not act intentionally (not q) if they are not directed by someone (not d). [conclusion] Then someone has tampered with the spacecraft systems (d). And that is called a saboteur.

Note that the content of the argument is to discover the human hand, an intention, where there are not supposed to be. The same thing goes the argument of design, although that is not human hand, it is supposed to be personal.

At this point, we must stress the arguments [1] and [2] are used for information that can not be obtained by observation or from an informant. You can not see God running the world or they have not been able to observe the saboteur

But despite this, no one would doubt that given what we know, that is, the premises, the conclusion can not be dismissed as false. This is a very important aspect. Indeed, one could state that has not been taken into account all information, all the premises (some students do this.). We could, for example, think on chance. But neither of our two travellers does. And we note who draws the conclusion (Tom) agrees, for his manner of speaking, with Dick’s premise. It is therefore important to introduce the arguments in dialogue form.

Therefore, we have that in the above arguments the conclusion depends on the premises and the only premises we have taken into consideration. If there is new information or we had not taken into account any information, then our arguments would no longer be useful. But that does not mean that the arguments, given the premises, were bad arguments. They are useless, but would remain good deductions by assuming the premises are true and there is no more.

Second Stage. Similar arguments.

From here, we can go one step further and note that there are arguments that have nothing to do with the topic of intentions but have a similar structure. For this I propose the arguments [3] and [4].

[3]
Tom: Dick has put thieves to flight who wanted to steal from him. (q)
Harry: He's just not a coward. (Not r)
Tom: Although he is not a coward (not r), he could not put thieves to flight (not q) if he could not fight (not d).
Tom: You're right.

Implicit conclusion: Dick can fight (d).

A similar conversation:

[4]
Tom: Dick approved. (q)
Harry: Dick doesn't go out much. (Not r)
Tom: Although he doesn't go out much (not r) Dick can't approve (not q) if he doesn't study (no d).
Harry: That's true.

Implicit conclusion: Dick studies (d)

At this time, it is convenient to present [3] and [4] under the form of teleological argument.
A convenient **exercise** could be like the next. Fill a table: one argument, one column; one premise, one row. (It will be easier if the teacher fills some squares and the two first columns are arguments [1] and [2].)

Third Stage. Similarities Again.

Once students have seen the structural similarity of the arguments, the teacher will stress again that all the arguments are used for information that can not be obtained by observation or from an informant. You can not see God running the world, they have not been able to observe the saboteur, and we have not been able to see if Dick spends hours at home playing or studying.

But despite this. We have certain premises. These premises are the only relevant. And those premises are assumed true. And if all this is so, the conclusion will be true. Moreover, it have to be true. This can be reformulated by saying that the reasoning in question, if we consider that the premises are true then the truth of the conclusion is warranted.

This is something that will seem intuitive to students. The question is whether it is justified. And indeed it can be shown that it is. The way is to use a truth table of the formulas involved in reasoning.

Fourth Stage. Truth Combinations.

Now, as we have just seen, the goal is justify an intuition by truth tables. Again it shouldn't explain in a formal way what is a truth table. It is enough for the students that all the possible combinations of truth and falsity of the formulas involved are occurring. It is worth stressing the significance of this part of the process. The teacher knows that he is building a truth table and applies the semantic definition of validity: if the premises are true (at once), the conclusion also will be, in every case. But the student only applies some ideas which come from the previous examples.

The difficulty here is that one of the premises is complex. Now propositional variables should be introduced (only as proposition names) and showed that the four arguments have the same form. This is easy except in premise 3.

We will begin showing the identity of syntax of the premise 3. I use grammatical categories already known by students. Premise 3 is a compound proposition. Logically, the truth value of a compound proposition depends on the simple ones and logical propositional-operators expressed by certain words.

The teacher focus the question on the truth (Remember: we introduced arguments to get information). The logic studies the combination and transmission of truth between propositions. And there are words that are used for the matter.

For example, I begin with negation and ask students to truth value of a negative proposition. The answers are easy and clear.

Afterwards I remember conjunctive compound propositions and disjunctive (exclusive) compound propositions. And I ask students about truth conditions of this propositions. Again, the answers are the corrects one. Obviously, the result is that the truth or falsity of a compound proposition depends on the truth or falsity of the simple propositions that compose it and certain words. And that words means certain true value given the true value of simple propositions.

We finish this stage summarizing the meaning of "and" and "or" (exclusive) by its well known truth tables and introducing the expression "if ... then ...". It is not necessary to go into details about the peculiarities of the conditional. This table expresses *some* uses of conditional "if ... then ..." and we say so to students. (Remember that they are not taking a course in logic).

Fifth Stage. Deduction.

At this point, we can deal with deduction: in all cases in which the premises are true, the truth of the conclusion is warranted. That is the student's intuition. And we will apply now.

The relevant propositions are:

q: natural bodies act intentionally.
r: natural bodies have intelligence.
d: someone directs natural bodies to an end.

The reasoning would be:

[premise 1] q
[premise 2] not r
[premise 3] If not r then (if not d then not q).
[conclusion] Therefore d.

The truth table is:

r	*q [premise 1]*	*d [conclusion]*	*not r [premise 2]*	*not q*	*not d*	*if not d then not q*	*if not r then (if not d then not q) [premise 3]*
T	T	T	F	F	F	T	T
T	T	F	F	F	T	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	T	T	T
F	**T**	**T***	**T**	F	F	T	**T**
F	T	F	T	F	T	F	F
F	F	T	T	T	F	T	T
F	F	F	T	T	T	T	T

There is only one case in which the premises, all at once, are T (in bold). And indeed, in this case, the conclusion is true (starred).

We have finished but it is advisable to sum up what has been done and how it was done.