

## IS IT EASY TO LEARN THE LOGIC?

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### 1. Resume

This paper contains the topics that generate the greatest difficulties in teaching the first-order logic in the human sciences, especially the problem of interpretation and symbolization of natural language, also, how logical rules are generated from the truth-functions, the distinction between the use of demonstration and decision-making procedures in the validity analysis of the arguments and the foundation's importance of the principles of classical logic in the context of laws and rules of logic.

### 2. Introduction

Logic, from Aristotle to our days, is the science of inference or argument. Logic distinguishes when an argument is correct or not. In a correct argument, the deduction is guaranteed to result from a variety of reasons. Since the analysis of correctness of the arguments is a central theme in the teaching of logic, I will sustain the difficulties encountered in teaching students in the humanities, specifically in undergraduate college. Particularly, I will outline some basic issues of classical logic, which require particular operations, and in some cases need ontologic foundation to distinguish between conceptual categories. In this context, I will briefly deal with the problem of interpretation and symbolization of arguments, truth tables seen as the ontology of logical rules, the importance of the decision procedures for propositional formulas and first grade monadic predicate logic. I also find very important to comment on the ontological basis of logical principles to understand the reason of the laws and rules of logic.

### 3. Logical interpretation and symbolization of natural language

Text understanding for a logical interpretation requires knowing the basics of logic, simple propositions and connectors, and if it is an argument, to distinguish the conclusion from a set of premises. In the symbolization it is necessary to grasp the logical sense of the text, which should not be separated from its semantic content, so the symbolism capture the syntax and meaning of the text. For example, given the following texts<sup>1</sup>:

1. If a test actually measures reasoning, it cannot be too difficult because, if so, no one could solve it, nor is it too easy, as everyone would answer it.
2. God forgives, time does not, hence only the strong wil survive.

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<sup>1</sup> "If a test that actually measures the reasoning = p", "The test is too hard = q", "no one could solve test = r", "The test is too easy = s", "everyone would answer test = t". With these simple propositions can reconstruct the text using the sequence that indicates the formal structure.

For a logic student, the problem that appears at first sight in the text 1 is the lack of syntax clarity to be symbolized in propositional logic (PL); in other words, it is difficult for him to construct the following formal structure which corresponds to that text:

If q then r, and if s then t; therefore, if p, then not q and not s.

$$[(q \rightarrow r) \wedge (s \rightarrow t)] \rightarrow [p \rightarrow (\sim q \wedge \sim s)]$$

If the text appeared as just described in the formal structure, the symbolism would be simpler. Now, regarding text 2, a PL symbolization would not capture more accurately the meaning of the proposition, but if we interpret it in the predicate logic, we find that the terms 'God' and 'time' are not identifiable. Therefore the symbolization requires the interpretation of PL and first order predicate logic, as shown below:

$$r \rightarrow (p \wedge \sim q)^2 \text{ is not the same as } (\forall x)(Sx \rightarrow Fx) \rightarrow (p \wedge \sim q)^3$$

As we can see in the above examples, the symbolization of natural language is not mechanical because in their syntactic and semantic construction the logic elements generally appear implicitly.

Regarding this problem, a logic student should be subjected to a constant field work, consisting of readings from all kinds of texts, literary, philosophical, scientific, etc., so that he become familiar with the construction of simple propositions, the recognition of the logical elements, especially in the construction of the formal structure of language texts.

#### 4. Truth functions and logical rules

My teaching experience has allowed me to observe that students do not tend to link the conceptual categories with logical operations. They often memorize mechanically the truth tables for each propositional operators, but when running the operations have no clear idea of the reasons of the truth-functions of each operator.

To describe the problem of the relationship between the truth charts and the logical rules, we introduce the following quite trivial definitions: If 'A' is a formula<sup>4</sup>, then the truth of 'A' or the falsity of 'A' is as follows:

$$V(A) = \text{Def. } A$$

$$F(A) = \text{Def. } \sim A$$

<sup>2</sup> God forgives = p" "time do not forgives =-q", "only the strong survive = r."

<sup>3</sup> 'Fx = x is strong, 'Sx = x survive'

<sup>4</sup> The capital letters A, B, are metavariables representing formulas. In this case we are defining the truth table for denial.

From this perspective,  $(A \wedge B)$  is assumed to be true, and  $\sim (A \wedge B)$  is assumed to be false, then each of the formulas can be transformed into equivalent using truth tables, as follows:

$$1. (A \wedge B) = \text{def. } \sim (\sim A \vee \sim B)$$

This definition interprets the “conjunction A and B is true” if and only “it is not the case that A is false or B is false” because the conjunction rule says “A and B is true only when both their components are true, and is false when at least one of its components is false”. So, for the false conjunction one has the following definition:

$$2. \sim (A \wedge B) = \text{def. } (\sim A \vee \sim B)$$

Following this same procedure we get the following definitions for disjunction:

$$3. (A \vee B) = \text{def. } \sim (\sim A \wedge \sim B)$$

$$4. \sim (A \vee B) = \text{def. } (\sim A \wedge \sim B)$$

These four definitions are known as the “De Morgan’s Laws”. With respect to the conditional the rule says: “if A, then B” is true, when either the antecedent is false or the consequent is true, and “if A then B” is false when the antecedent is true and the consequent is false”. Using this rule one has the following definitions known as the Laws of implication<sup>5</sup>.

$$5. (A \rightarrow B) = \text{def. } (\sim A \vee B)$$

$$6. \sim (A \rightarrow B) = \text{def. } (A \wedge \sim B)$$

I think it enough to point out, as examples, the difficulties of students concerning the management of equivalences from the truth tables, and the interpretation and application of the implication rules like the Modus Ponens:

$$\begin{array}{l} A \rightarrow B \\ A \\ \hline \therefore B \end{array}$$

Describing the interpretation of the truth tables in this rule, it can be observed that the first premise says: "A implies B" and the second says: 'A ' is true. Then to preserve the truth of the conditional, ' B ' is necessarily true. In this case, the conclusion is a *semantic consequence* of the premise set.

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<sup>5</sup> For our purposes, 'A implies B' connotes the real logical relationship between A and B, but 'if A then B' is only a formal relationship is not necessarily true. Similarly we can distinguish conceptually between 'A equals B' and 'A if and only if B'.



The presence of contradiction in all branches of the diagram tells us that the formula is valid or logically true.

For the formal proof of the show, we preferred to use the method of natural deduction Fitch<sup>10</sup> type; also the Anderson – Johnstone<sup>11</sup> version, and Mara Manzano's natural deduction<sup>12</sup> are well known.

Given that our formula (1) is valid, we demonstrate it by Fitch natural deduction type<sup>13</sup>, as follows:

$$(p \rightarrow q) \wedge (\sim q \vee r) \rightarrow (\sim p \vee r)$$

1	(p → q) ∧ (∼ q ∨ r) Hipotesis
2	(p → q) (1) Elimination ∧
3	(∼ q ∨ r) (1) Elimination ∧
4	∼ q Hipotesis
5	(p → q) (2) Reiteration
6	∼ p (4,5) Modus Tollens
7	(∼ p ∨ r) (6) Introduction ∨
8	r Hipotesis
9	(∼ p ∨ r) (8) Introduction ∨
10	(∼ p ∨ r) 3, (4-7) (8-9) Elimination ∨
11	(p → q) ∧ (∼ q ∨ r) → (∼ p ∨ r) (1 – 10) Introd. →

The importance of this demonstration, among other things, comes from the distinction between the levels from the main proof and the respective sub-proofs, the development of hypotheses and application from the rules to justify the successive sequences, widely used particularly philosophical reflection.

In my long experience as a professor of logic, I have observed that students, with some difficulty, do calculus operations, due to the difficulty that arises from the mechanical application of semantic diagrams, forgetting that these semantic diagrams have their roots in truth tables; in other words, the purely mechanical operations lead to not knowing the reasons for such

<sup>10</sup> Fitch, *Symbolic Logic*, pag. 20 and ss.

<sup>11</sup> Anderson, John y Henry W. Johnstone, *Natural deduction*. Pag. 3 and ss.

<sup>12</sup> Op. Cit., pag. 137 and ss.

<sup>13</sup> The introduction and elimination rules see Fitch, Op. Cit. Pages. Cap. II, also; María Manzano, op cit., pages. 159 to 161,

operations. The problem is still worse in Natural Deduction proofs because the effort of reasoning is richer in possible alternatives to obtain the desired result by applying logical rules. However, the logical operations are necessary both in the decision as in the proof, because they put in motion the mind to get the best possible level of abstraction of reasoning. Hence, we must insist on presenting problems, not just as exercises in logic formulas, but as field work where the reasoning in natural language can be analyzed, recommending careful reading in the various fields of knowledge.

## 6. The reason of the classic principles

Frequently one encounters questions like, what is the use of logical principles if they are not used operationally like the De Morgan's Laws or Modus Ponens? What is the importance of learning them and mention them? In colloquial language, saying "Mary studies at the Catholic University imply that Mary studies at the Catholic University", expresses the principle of identity. However, this expression to our common sense seems trivial, or is merely an expression of *petitio principii*. Obviously all the above refers to the foundation of logical principles. Therefore, I think it important that a student of logic has a clear idea about the role of logical principles in classical logic. The answer to that question leads us to the roots of Greek philosophy, specifically with reference to the parmenidean concept of "being".

Why the 'being'? Discussion about the 'being' is a complex subject like walking on a slippery slope, so the track leading to the formal origin of logical principles is the thought of Parmenides: 'What is' was not, nor will be, since it is entirely 'now'. Being is still, remains in its place, is in itself and by itself, since it lacks nothing, because if it lacked something, it would lack everything. This being is only intelligible and remains identical to itself, "is fully within its boundaries". In addition, in verses 1-4, it seems to anticipate the interpretation of the sign: the first sign is unbegotten and indestructible, and the second sign is "integral, unique"<sup>14</sup>. This interpretation is related to the timelessness of "being", because it denies the possibility of "what is" arises from "what is not", since it cannot be thought or talked about .

Our reasoning about Parmenides' thought allows us to intuitively interpret the 'being' as an 'entity', and the entity could attributed predicates, obtaining thus judgments or propositions. So, in relation to the thought of Parmenides, we can explain the ontology of the three principles of classical logic.

1. *The principle of identity*: "The being is" ontologically states that the "being" is identical only to itself. Truth of a proposition implies the truth of the proposition itself. Modern logic expresses it with symbols in different ways, here are some:

$$(p \rightarrow p) \text{ or also } (p \leftrightarrow p), \text{ or as } (A = A)$$

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<sup>14</sup> Mondolfo, R., *El pensamiento antiguo (V. I)*, page. 80

Now the question is what is the purpose of the thesis of identity? The importance of Parmenides is full as soon as the object itself does not change over time, is lasting, eternal in our intellectual conception. Mathematical entities, for instance, persist over time, we can see its timelessness in numbers, thus number two has been, is and will be just identical to itself. This principle is applicable to object identification, even though the object, as matter, has undergone many changes. Besides, in what concerns thought, the principle of identity leads us to think in the same direction. If the thesis from my statement is A, in the same context I cannot hold non-A, which means that holding A must not lead to contradiction.

2. *The principle of non contradiction*: "The being is" and "the being is not" is contradictory; then in our reason "it is impossible for the being to be and not to be at the same time". In Aristotelian logic, "it is impossible that a proposition is true and false at same time". According to Kant, it is the supreme principle of reason. In our minds, it is not possible that "p" is true and false at once. In classical logic it is stated as:

$$\sim (p \wedge \sim p) \quad \text{or also} \quad (\forall x) \sim (x \neq x)$$

3. *The principle of the excluded middle*: According to Parmenides, "either the 'being' is or the 'being' is not". In logic, a proposition is true or not true there being no third possibility. For instance, "either María studies or does not study in the Catholic University. Formally:

$$(p \vee \sim p)$$

Undoubtedly, the formulation of these three principles is the most important basis in developing formal sciences. Leibniz considered it fundamental the use of the principles; first, the *principle of identity* and *principle of non contradiction*, then, in order of importance, "the principle of the indiscernibles"<sup>15</sup> and "the principle of sufficient reason". On indiscernibility we can find the law that somehow appears in the Topics of Aristotle<sup>16</sup>, the *law of indiscernibility of identicals*<sup>17</sup>, which belongs to the predicate logic<sup>18</sup> of second order.

Consequently, if a student of Logic assimilates that the three principles mentioned are the ontological basis of bivalent logic, he would have the clear conception that all logical truths are derived from the three principles, and, at the same time, that any logical truth can be reduced to any the three logical principles. Any formal decision-making process can be effective for these purposes.

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<sup>15</sup>  $(\forall x)(\forall y)[(x = y) \rightarrow (Px \leftrightarrow Py)]$

<sup>16</sup> Deaño, A., *Introducción a la lógica formal*, page 262.

<sup>17</sup> *Ibid.*

<sup>18</sup> In the symbolic formulation of these laws, said Deaño, the verb "to be" is used in three ways, as synonymous with identity, as inclusion and as belonging, but symbolically has been assimilated, so it disappears.

## 7. Conclusions

7.1. The management of the conceptual categories of logic requires a careful and systematic analysis of texts from different areas of knowledge, analysis which should be directed to the construction of simple propositions, recognition of connectives and the corresponding formal structure for the symbolization.

7.2. The operations of logical rules must be supported by the rationale in each case to avoid mechanization, because most students will normally not take another course in Logic.

7.3. The exercises of decision-making procedures and natural deduction must meet the objective; in addition to logical problems solving, they will generate in the students mental agility training and will improve their level of abstraction.

7.4. The understanding of logical principles should lead the student to relate the rules of logic and to the reflection on the formulation of hypotheses, and methodology of scientific research.

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